TECHNICAL REPORT ARBRL-TR-02325

EXTREME VALUE QUANTAL RESPONSE EXPERIMENTAL DESIGN

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May 1981

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER TECHNICAL REPORT ARBRL-TR-02325	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) EXTREME VALUE QUANTAL RESPONSE EXPERIMENTAL DESIGN		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Jill H. Smith Jerry Thomas		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Ballistic Research Laboratory ATTN: DRDAR-BLB Aberdeen Proving Ground, MD 21005		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS RDT&E 1L162618AH80
11. CONTROLLING OFFICE NAME AND ADDRESS USAARRADCOM Ballistic Research Laboratory ATTN: DRDAR-BL Aberdeen Proving Ground, MD 21005		12. REPORT DATE MAY 1981 13. NUMBER OF PAGES 30
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Quantal Response
Experimental Design
Extreme Value Design
Alexander Extreme Value Design
Method of Reversals
Maximum Likelihood Estimates
Sensitivity Testing
Shielding Thickness
Round-to-Round Propagation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (eal)

An experimental design has been developed to be used to determine the shielding thickness required between rounds stored in a storage area to prevent round-to-round propagation from an initial explosion. Extreme value quantal response techniques were used with shielding thickness as the stimulus variable. The developed design drastically reduces the sample size required for a given probability and confidence when compared with known distribution-free extreme value designs.

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TABLE OF CONTENTS

I. INTRODUCTION

The Terminal Ballistics Division of the Ballistic Research Laboratory encountered the problem of determining how thick the shielding should be between rounds of ammunition stored in a storage area to prevent round-to-round propagation from an initial explosion. Vulnerability analysis indicated that the probability of survival of the storage area would drastically decrease with an increase in the number of rounds exploding. Prior testing has shown that shielding material placed between rounds could prevent neighboring rounds from exploding. Due to space limitations in the storage area, it was desired to keep the shielding thickness to a minimum and simultaneously minimize the probability of round-to-round propagation from the initial explosion.

It was decided that the specific objective of the test would be to find the shielding thickness needed to be 90% confident that the probability of a neighboring round exploding is less than 0.1.

The problem appeared to fit into the category of extreme value quantal response problems. The stimulus is characterized by a variable X, in this case the thickness of the shielding which affects the stimulus, and the probability of a response associated with a given X is described by a nonresponse function M(X). (Usual notation has M(X) as the probability of response. However, defining M(X) as a nonresponse is more natural for this problem.) This function is assumed to be nondecreasing with increasing stimulus levels.

A discussion of available designs and the modified design chosen for the experiment is contained in the following chapters.

II. AVAILABLE DESIGNS

A nonparametric approach was taken because of the lack of information about the response function. For a given probability of an explosion, α, where $0 < \alpha < 1$, we wish to determine a value, X_α, such that $M(X_\alpha) = 1 - \alpha$. As stated, the probability in which we are interested is $\alpha = .10$, and therefore is in the tail of the response distribution. From a review of the available designs in the literature the only nonparametric test designs available for testing in the tail regions are the Alexander Extreme Value Design and the Rothman Design. Of these, the Alexander Extreme Value Design is preferred since it:

1) is "generally more efficient than other available nonparametric designs, and is asymptotically as efficient as the best parametric stochastic approximation when distributional assumptions are valid,"[1]

2) has significantly simpler design rules and analysis procedures than the Rothman Design, and

3) does not differ from the Rothman Design in median required sample size.

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[1] D. Rothman, M. J. Alexander and J. M. Zimmerman, The Design and Analysis of Sensitivity Experiments, NASA CR-62026, Vol. I, p. 74.

III. ALEXANDER EXTREME VALUE DESIGN

The Alexander Extreme Value Design assumes only a nondecreasing response function as the stimulus increases.

A. Design Rules

1) The first test is at shielding thickness level X_1, the a priori best guess of X_α.

2) Testing is performed by alternately increasing and decreasing sequences of test levels. The test levels are increased or decreased by a step size δ, where δ is an estimate of $M'(X_\alpha)$. Terms such as "higher" and "level above" refer to thicker shielding levels, and "below" and "lowest" refer respectively to thinner and thinnest shielding thickness levels.

3) The first sequence decreases the levels until a response (explosion) is observed.

4) The first test of an increasing sequence is at the level above the highest level at which a response has been observed. The increasing sequence ends at level X_i such that in the corresponding zero region* less than or equal to X_i at least N nonresponses have been observed. Values for N can be found from

$$(1 - \alpha)^n = 1 - P \tag{1}$$

where $N = [n] + 1$ and P is some specified probability.

5) The first test of a decreasing sequence is at the level above the highest level at which a response has been observed. If the result is a response, the sequence ends; otherwise, one more test at the next lower level is performed.

6) Testing terminates when there are three adjacent levels, X_r, $X_r + \delta$ and $X_r + 2\delta$ such that at least one response has been observed at X_r and none at a higher level, and a total of N nonresponses have been observed at $X_r + \delta$ and $X_r + 2\delta$. (δ is the step size between levels.)

7) The maximum likelihood estimate of X_α, $\hat{X}_\alpha$, is found by the method of reversals and linear interpolation (see Appendix).

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*Zero region - stimulus region above the highest level at which a response has been observed.

B. Analysis

We are interested in the $\alpha = .1$ quantile of the response distribution, that is, the value, $X_{.1}$, at which the probability of a response is .1. Therefore, the probability of a nonresponse at the $X_{.1}$ quantile is $(1 - .1)$. The probability of n nonresponses, assuming the n tests are independent, is $(1 - .1)^n$. The probability of at least one response out of n tests is $1 - (1 - .1)^n$. Specifying the probability of at least one response out of n tests at the $X_{.1}$ quantile to be $P = .9$, we have

$$.9 = 1 - (1 - .1)^n.$$

This, with a slight algebraic manipulation, is Equation (1) with $\alpha = .1$ and $P = .9$. Solving, $N = [n] + 1 = 22$. Hence, we would expect with probability .9 at least one response out of 22 tests at the $X_{.1}$ quantile. If we observe 22 nonresponses at some level X_*, we can assume we are not at the $X_{.1}$ quantile and, in fact, the

$$\text{Prob}\ \{X_{.1} < X_*\} > .9\ .$$

Using the above argument, we can conclude from the Alexander Extreme Value Design that the level at which the true probability of response is .1 is less than $X_r + 2\delta$ with ninety percent confidence. The point estimate of the $X_{.1}$ quantile can be found using the method of reversals outlined in the Appendix.

C. Simulation

Based on engineering estimates (judgement) for $X_{.5}$ and $X_{.75}$, a response distribution was hypothesized with which to Monte Carlo the Alexander Extreme Value Design for $\alpha = .1$ and $P = .9$. The response distribution assumed was the cumulative normal distribution with mean = .5 and variance = .14. (Note, however, that the test design and analysis procedures are distribution-free.) The smallest practical step size of shielding thickness was 1/8 inch.

Figures 1 and 2 are examples of the Alexander Extreme Value Design Monte Carloed to illustrate the design rules. Responses are denoted by "X" 's and nonresponses by "O" 's. I_i denotes the i-th increasing sequence and D_j the j-th decreasing sequence. The number of rounds required (NR), the maximum likelihood estimate of the .1 quantile $(\hat{X}_{.1})$, and the $(X_r + 2\delta)$ level are given for each simulation.

Figure 3 shows the distribution of the number of rounds required for 500 simulations of the above design. The number of rounds required is twice the number of responses and nonresponses shown for each simulation since a donor round must be detonated for each test round. The average number of rounds required to complete the test was 166, the median was 164 and ten percent of the tests required 184 rounds or more.

The distribution of the maximum likelihood estimates of $X_{.1}$ for the 500 simulations is given by the histogram in Figure 4. The distribution of $\hat{X}_{.1}$ is asymptotically normal about the true $X_{.1}$ quantile = 7.83. The distribution generated by the test data shown in Figure 4 has a mean of 7.77, which is in good agreement for 500 simulations, and is approximately normally distributed as shown by the overlying normal curve.

Figure 5 shows the distribution of level $X_r + 2\delta$ for 500 simulations. This is the level about which we can conclude that the

$$\text{Prob}\ \{X_{.1} < X_r + 2\delta\} > .9.$$

IV. MODIFICATION OF THE ALEXANDER EXTREME VALUE DESIGN

The median number of rounds required for the Alexander Extreme Value Design (EVD), as described in the previous section, was 164 as determined by the 500 simulations. Since the number of rounds available for testing was considerably smaller, the major objective in modifying the Alexander EVD was to reduce the number of rounds required, while maintaining the confidence level and the ability to compute the point estimate of the $X_{.1}$ quantile.

The Alexander EVD requires that a donor round be detonated for each test. The number of donors needed can be reduced by using one donor to detonate up to four test rounds (acceptors). Figure 6 shows the configuration of four acceptors per donor. Steel shielding will be placed between acceptors, as shown by the dotted lines, if interaction between acceptors is observed. Optimizing the number of acceptors per donor in the Alexander EVD reduces the number of rounds required by approximately 33 percent.

It was noticed that the rounds above level $X_r + 2\delta$ were used neither to establish the confidence statement, nor to terminate the test design, nor to compute the point estimate of the $X_{.1}$ quantile. By limiting each increasing sequence above the highest stimulus level at which a response has been observed, the rounds "wasted" above level $X_r + 2\delta$ can be eliminated. There is a trade-off in eliminating these rounds since the level $X_r + 2\delta$ can change if a response is observed at a higher level. Therefore, some testing should be above $X_r + 2\delta$ until more than half the number of rounds required to demonstrate the chosen probability are at levels $X_r + \delta$ and $X_r + 2\delta$. Testing at X_r and below is used in the determination of the point estimate of $X_{.1}$.

The following test design is the result of many Monte-Carlo simulations in which different starting levels, number of acceptors per donor and sequences of testing have been tried in order to minimize the required number of rounds, yet retain the confidence level and point estimate of the $X_{.1}$ quantile.

A. Modified Design Rules

1) The first test level is X_1, the best a priori guess of X_α. δ is the step size between levels.

2) One acceptor per donor is used, in a decreasing sequence, until a response is observed. Let X_r be the highest level at which a response is observed.

3) After the first response, the number of acceptors per donor in each test is increased to alternately three and then four. After the first response, three acceptors per donor are tested at the next three levels above X_r. Then four acceptors per donor having shielding at levels X_r and the next three higher levels are tested.

4) If another response is observed at a higher level, it becomes X_r, and testing continues alternating three and then four acceptors per donor until at least 12 (more than half the required 22) nonresponses have been observed at the two levels immediately above X_r.

5) When at least 12 nonresponses have occured at $X_r + \delta$ and $X_r + 2\delta$, the number of acceptors per donor is reduced to alternately two above X_r and then three, starting at X_r, for the remainder of the test.

6) Testing terminates when at least N (22) nonresponses have been observed at the two levels immediately above the highest level at which a response has been observed.

B. Analysis of the Modified Design

As in the Alexander Extreme Value Design, we have N = 22 nonresponses at $X_r + \delta$ and $X_r + 2\delta$ and can conclude that we are not at level the $X_{.1}$ quantile and in fact,

$$\text{Prob}\ \{X_{.1} < X_r + 2\delta\} > .9.$$

The point estimate can again be found using the method of reversals. Therefore, the changes in the test design have not affected the confidence statement or the point estimate.

C. Simulations Using Modified Design

Using the same response distribution that was used when simulating the Alexander Extreme Value Design, 500 simulations of the modified design were also Monte-Carloed.

Figures 7 and 8 are examples of the modified test design illustrating the modified design rules. Again, responses are denoted by "X" 's and nonresponses by "O" 's. The abscissa represents individual tests rather than sequences of tests as shown in the Alexander Extreme Value Design.

Figure 9 shows the distribution of the required number of rounds for the 500 simulations. The median number of rounds required was 67 and the mean number of rounds, 70. Only ten percent of the simulations required 93 or more rounds.

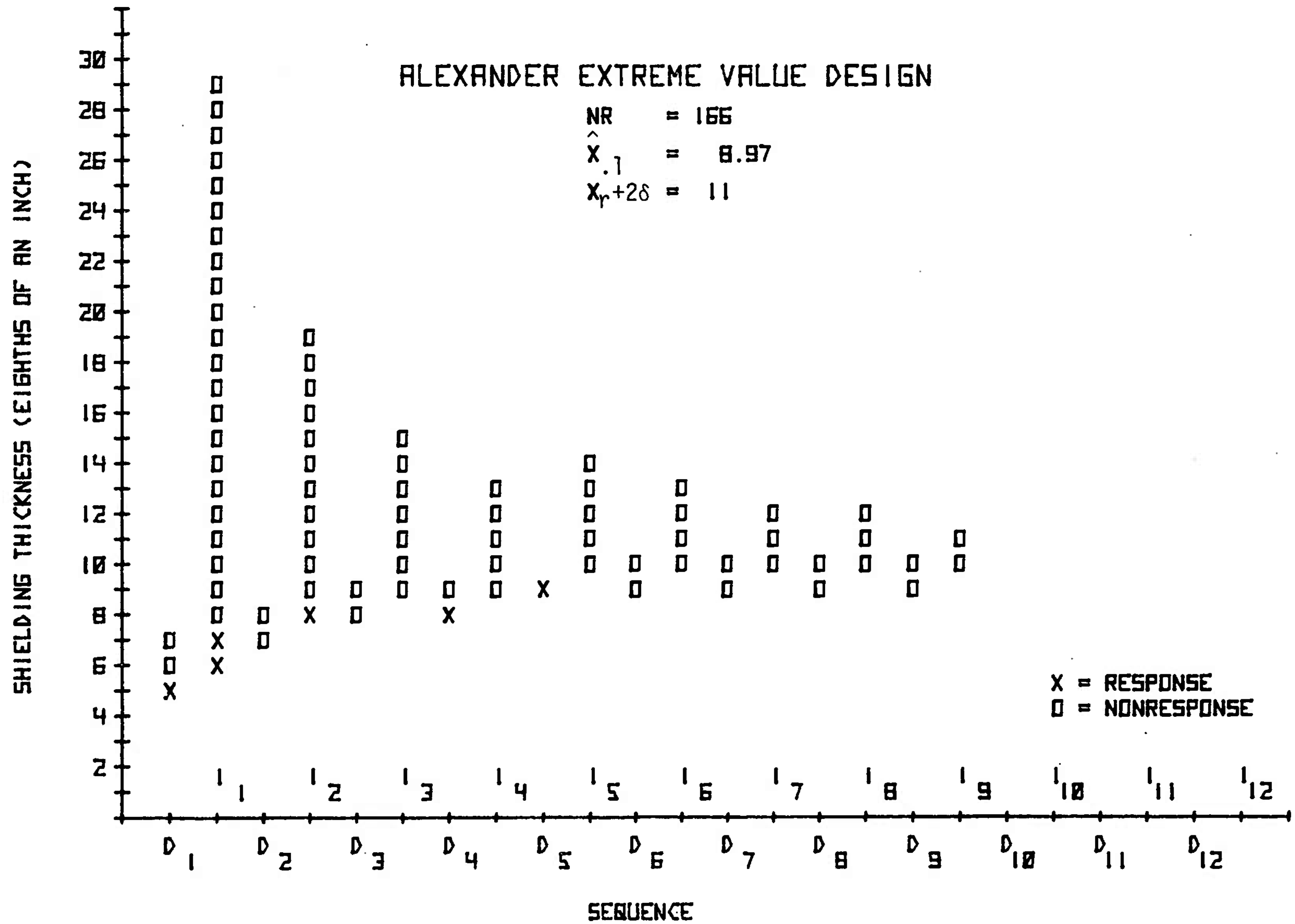
The histogram in Figure 10 is the distribution of the maximum likelihood estimates of $X_{.1}$ for the 500 simulations. Again, the distribution of the maximum likelihood estimates are asymptotically normal about the true $X_{.1}$ quantile = 7.83. The distribution shown has a mean of 8.00, and is approximately normally distributed as shown by the overlying normal curve. Figure 12 shows the distribution of number of rounds required for both the Alexander EVD and the Modified Alexander EVD. The Modified Alexander EVD is on the left and the Alexander EVD is on the right.

V. OTHER SIMULATION DISTRIBUTIONS

All the work thus far has used a normal distribution as the underlying distribution for generating the simulated data. In order to investigate the effect of different distributions on the two designs, the $X_{.5}$ and $X_{.75}$ "guestimates" were used to approximate three more distributions, the exponential, the uniform and the gamma. The results are summarized in Table 1. It should be noted that the true $X_{.1}$ quantile is different for each distribution. However, both the Alexander and Modified Alexander EVD give good estimates of $X_{.1}$. The Modified Alexander EVD average estimates of $X_{.1}$ are slightly higher than the Alexander EVD in all cases except for the uniform distribution. The Modified Alexander EVD requires less than half as many samples as the Alexander EVD for all the distributions. The average of the 90% upper confidence limit is also given for each distribution. The ratio of the mean sample size required for the modified Alexander EVD to the mean of the Alexander EVD is given in the last column. This ratio indicates that on the average the modified design requires 50% or less samples than the Alexander EVD.

VI. SUMMARY

The Alexander EVD was modified, mainly, by using multiple rounds per test and by limiting the number of rounds above the highest response. These changes resulted in a design that required less than half the rounds of the Alexander EVD in the simulations performed, regardless of the simulated distribution. The sample size required for Modified Alexander EVD ranged from 37% to 50% of that required for the Alexander EVD. The Modified Alexander EVD has simple design rules that permit the estimation of an extreme value of a quantile response function and the associated confidence interval. It should be remembered that neither the experimental design nor the analysis methods require the assumption of a response distribution. The design is distribution-free.



ALEXANDER EXTREME VALUE DESIGN
NR = 166
$\hat{X}_{.1}$ = 8.97
$X_r+2\delta$ = 11
X = RESPONSE
O = NONRESPONSE
SHIELDING THICKNESS (EIGHTHS OF AN INCH)
SEQUENCE

FIGURE 1

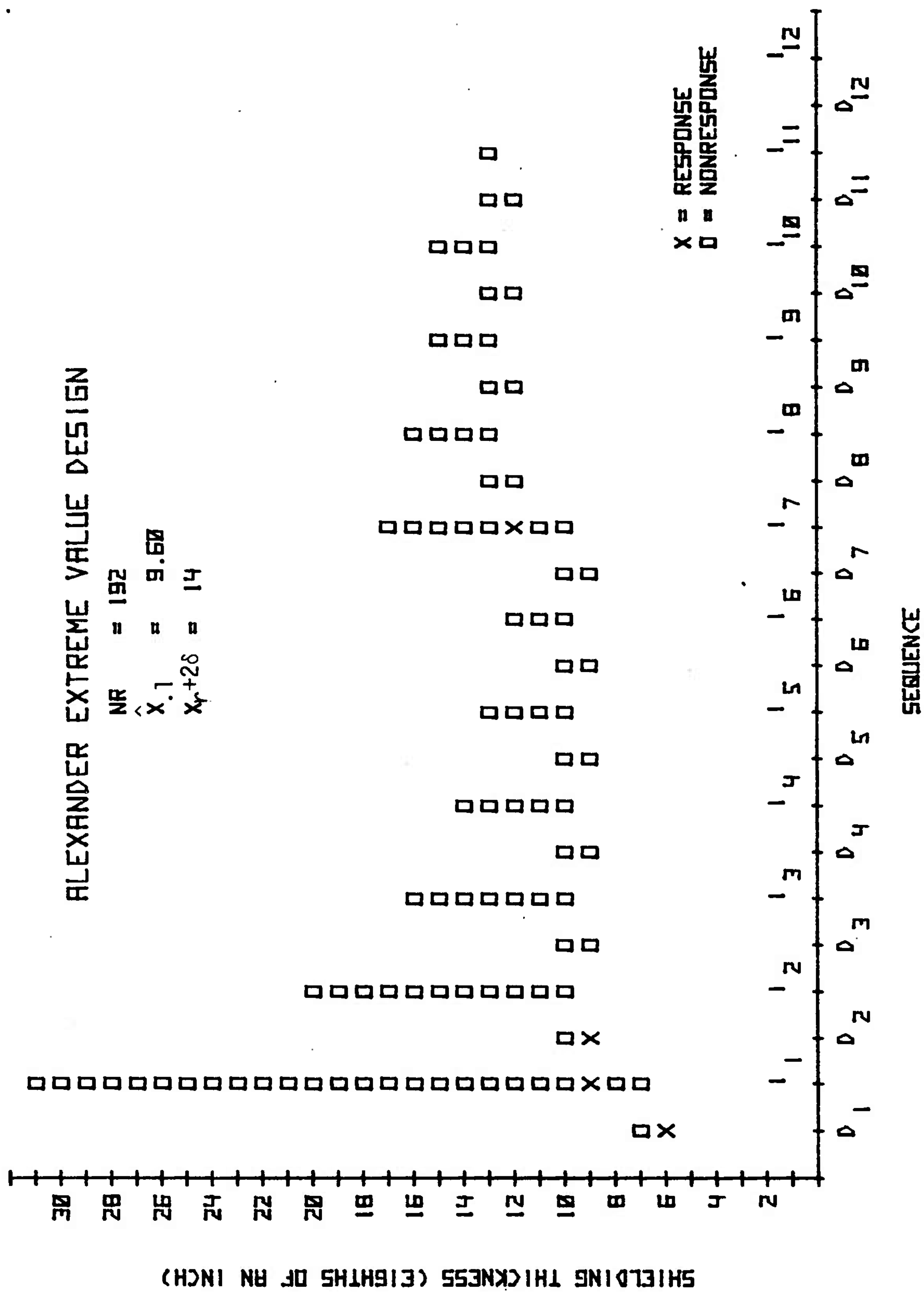

ALEXANDER EXTREME VALUE DESIGN
NR = 192
$\hat{X}_{.1}$ = 9.60
$X_{.1}+2\delta$ = 14
X = RESPONSE
□ = NONRESPONSE
SEQUENCE
SHIELDING THICKNESS (EIGHTHS OF AN INCH)
D_1 I_1 D_2 I_2 D_3 I_3 D_4 I_4 D_5 I_5 D_6 I_6 D_7 I_7 D_8 I_8 D_9 I_9 D_{10} I_{10} D_{11} I_{11} D_{12} I_{12}
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

FIGURE 2

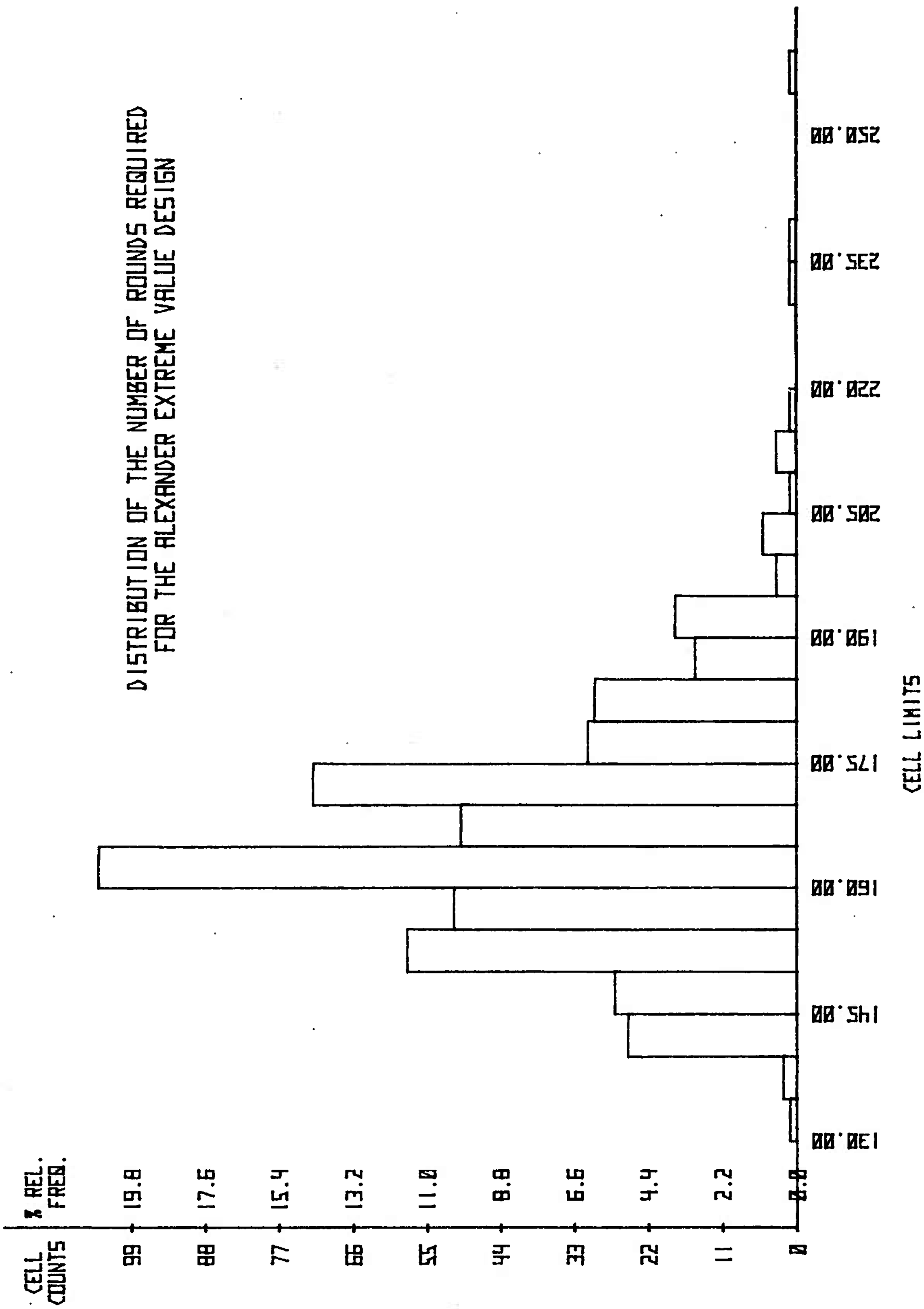

DISTRIBUTION OF THE NUMBER OF ROUNDS REQUIRED
FOR THE ALEXANDER EXTREME VALUE DESIGN
CELL COUNTS
% REL. FREQ.
0 11 22 33 44 55 66 77 88 99
0.0 2.2 4.4 6.6 8.8 11.0 13.2 15.4 17.6 19.8
130.00
145.00
160.00
175.00
190.00
205.00
220.00
235.00
250.00
CELL LIMITS

FIGURE 3

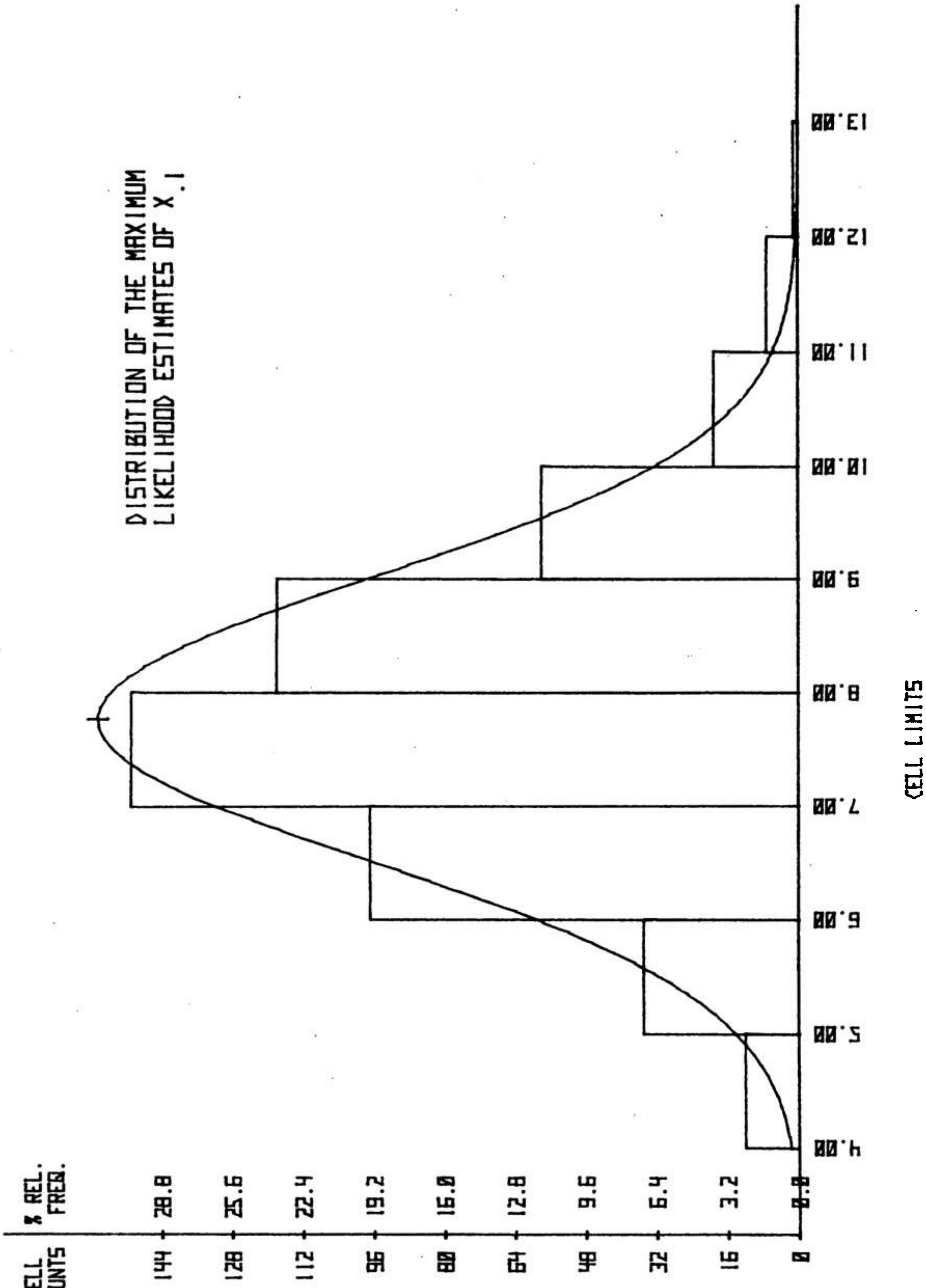

DISTRIBUTION OF THE MAXIMUM
LIKELIHOOD ESTIMATES OF X .1
CELL COUNTS
% REL. FREQ.
144
128
112
96
80
64
48
32
16
0
28.8
25.6
22.4
19.2
16.0
12.8
9.6
6.4
3.2
0.0
4.00
5.00
6.00
7.00
8.00
9.00
10.00
11.00
12.00
13.00
CELL LIMITS

FIGURE 4

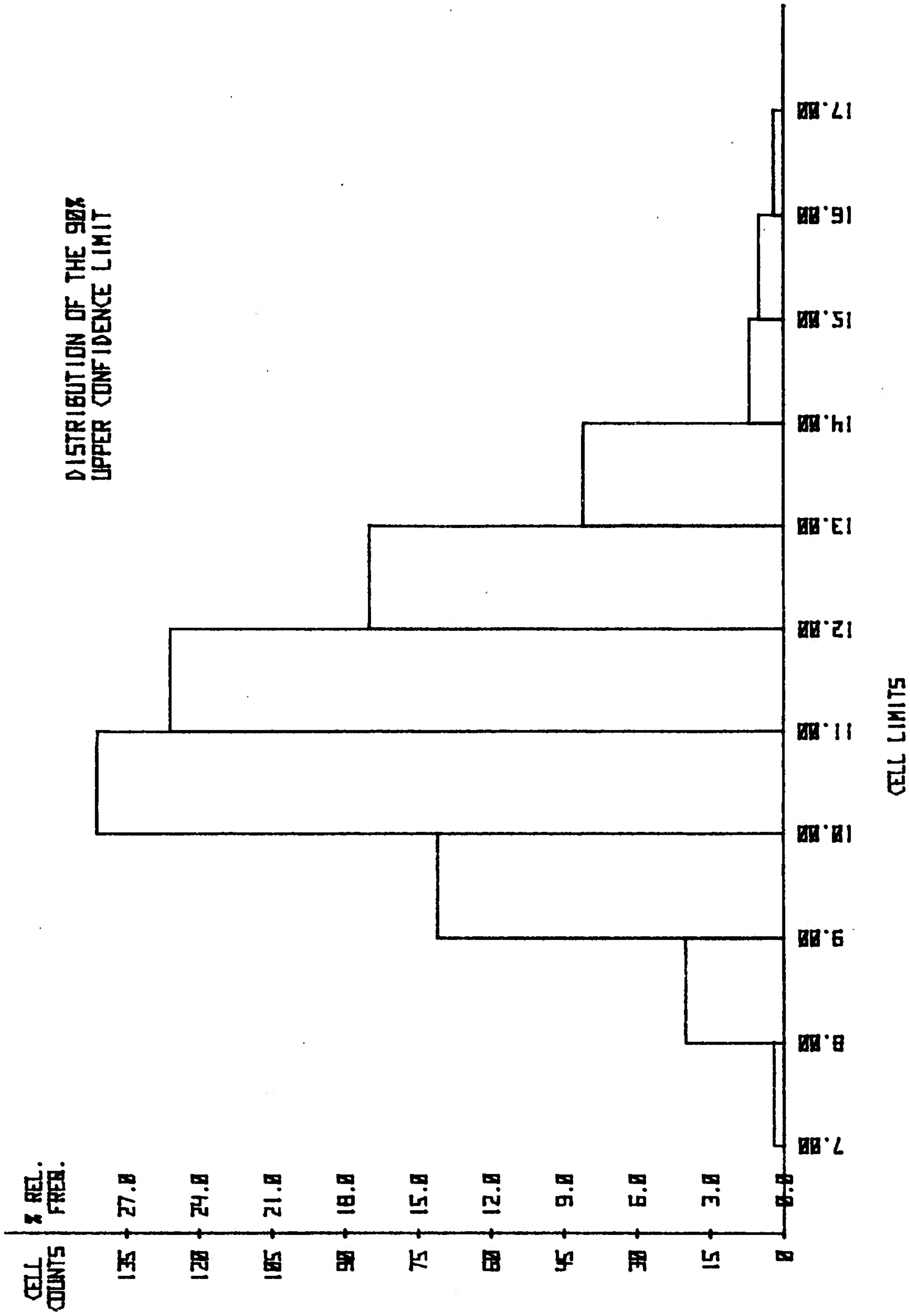

DISTRIBUTION OF THE 90%
UPPER CONFIDENCE LIMIT
CELL COUNTS
% REL. FREQ.
135 27.0
120 24.0
105 21.0
90 18.0
75 15.0
60 12.0
45 9.0
30 6.0
15 3.0
0 0.0
7.00
8.00
9.00
10.00
11.00
12.00
13.00
14.00
15.00
16.00
17.00
CELL LIMITS

FIGURE 5

MODIFIED TEST CONFIGURATION

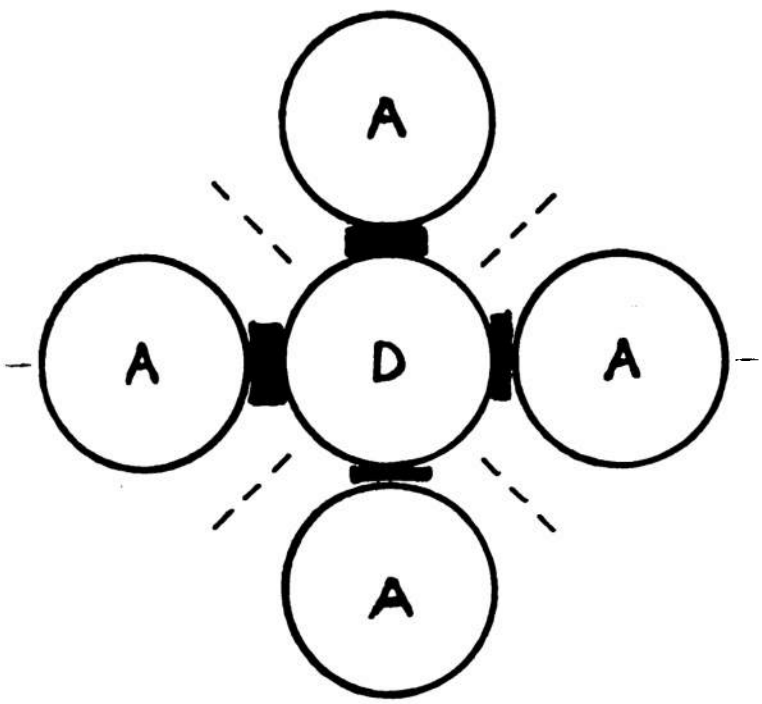


FIGURE 6

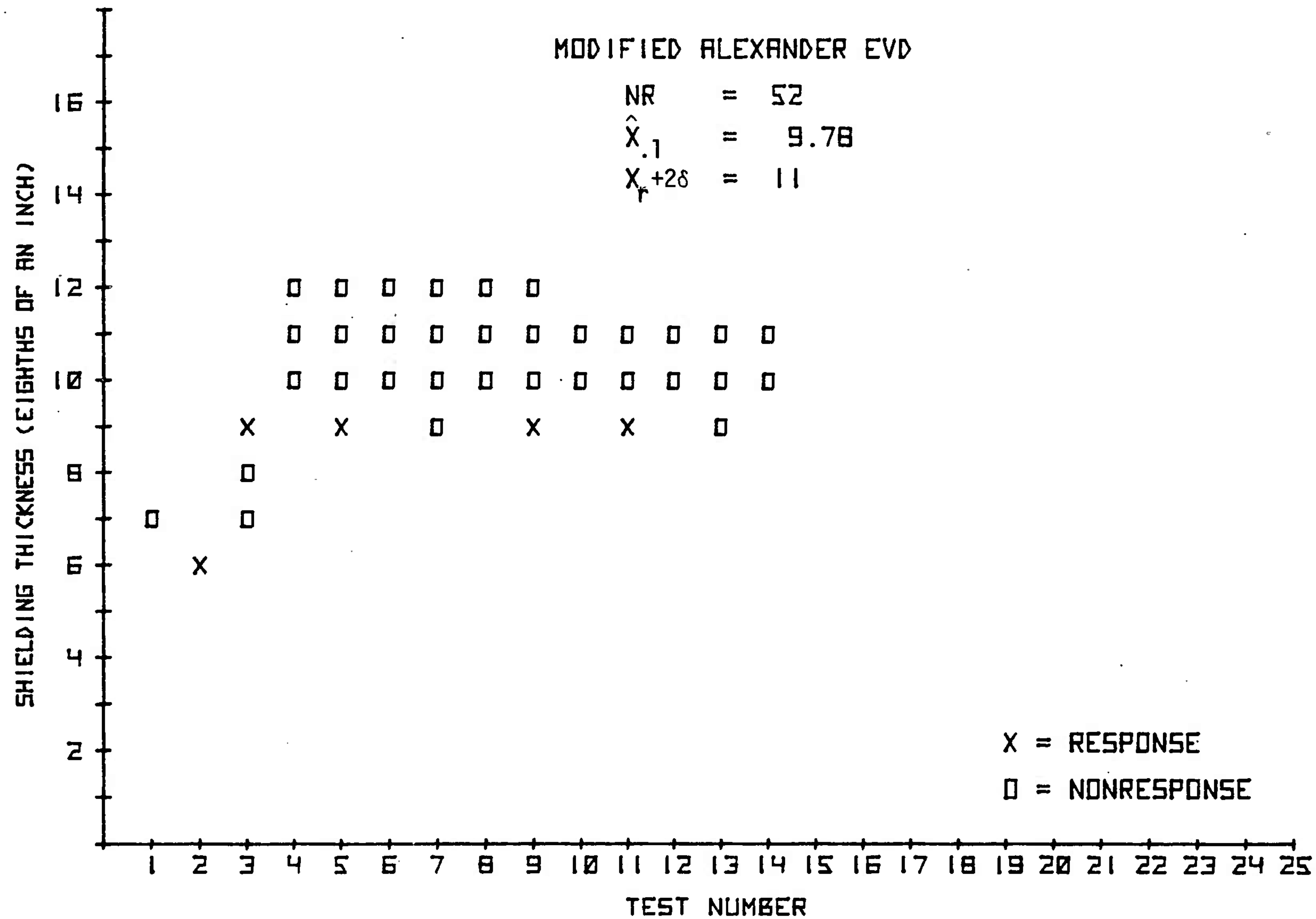

MODIFIED ALEXANDER EVD
NR = 52
$\hat{X}_{.1}$ = 9.78
$X_r+2\delta$ = 11
X = RESPONSE
O = NONRESPONSE
SHIELDING THICKNESS (EIGHTHS OF AN INCH)
TEST NUMBER

FIGURE 7

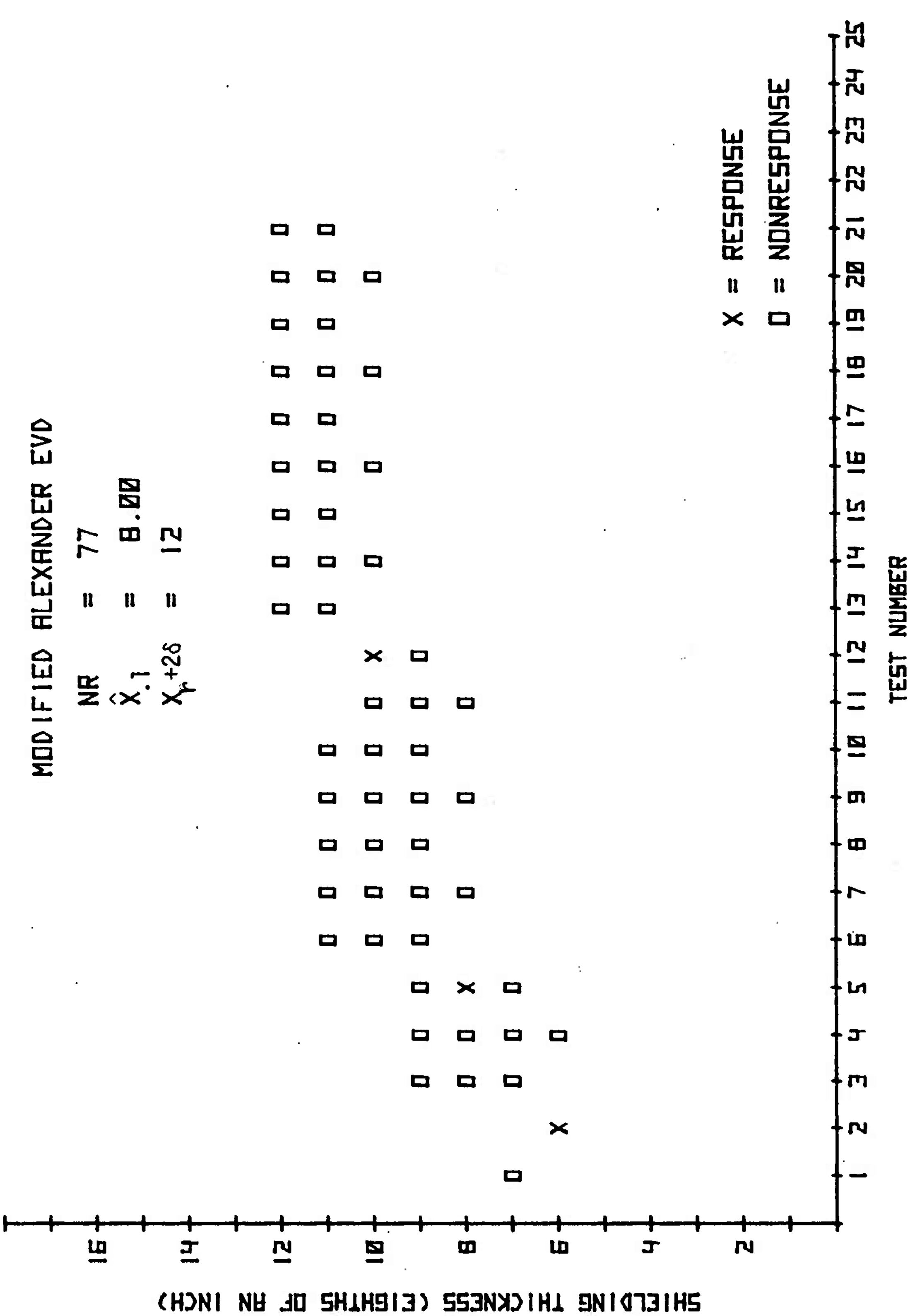

MODIFIED ALEXANDER EVD
NR = 77
X̂.1 = 8.00
X r+2δ = 12
X = RESPONSE
□ = NONRESPONSE
TEST NUMBER
SHIELDING THICKNESS (EIGHTHS OF AN INCH)
2 4 6 8 10 12 14 16
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

FIGURE 8

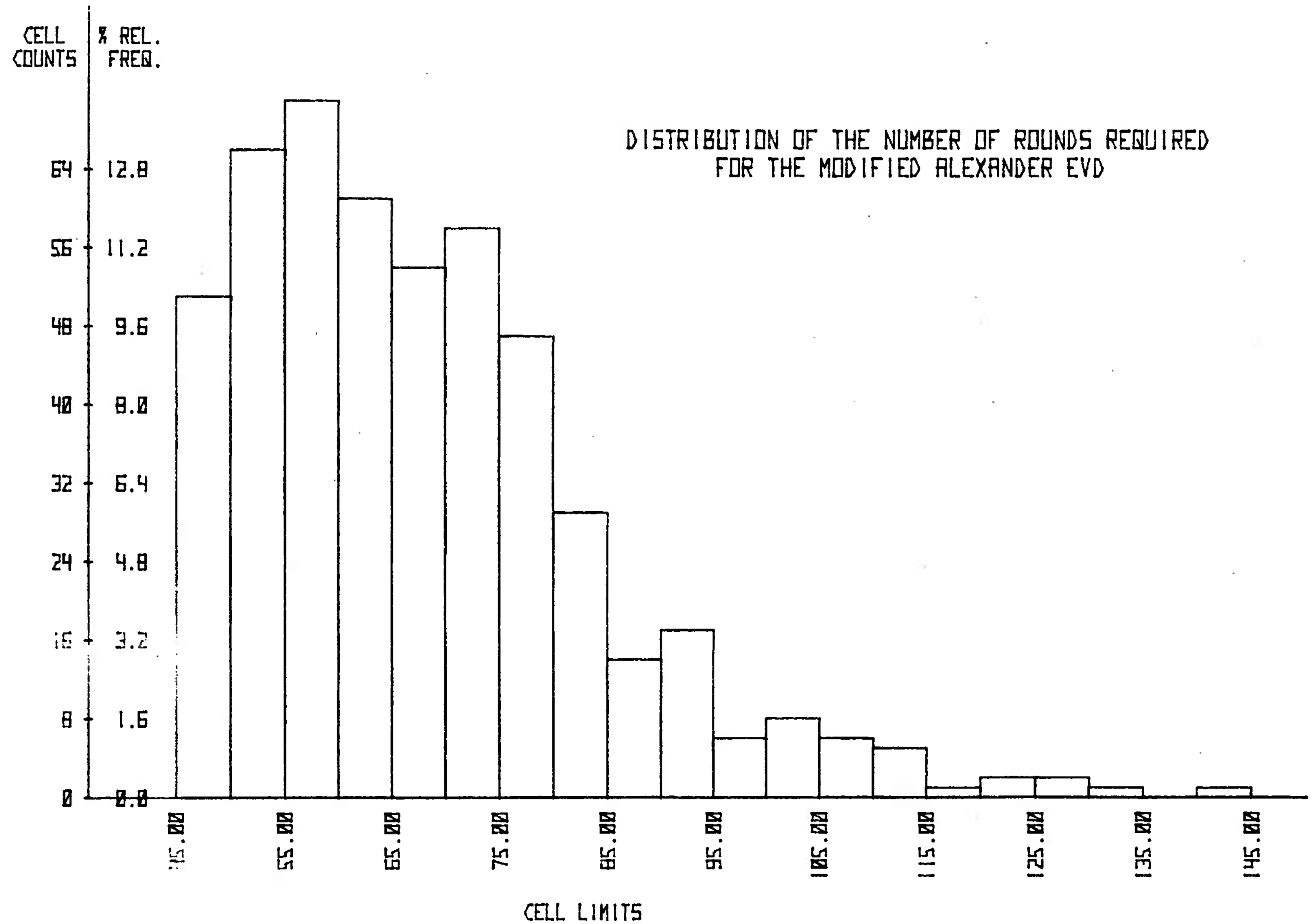
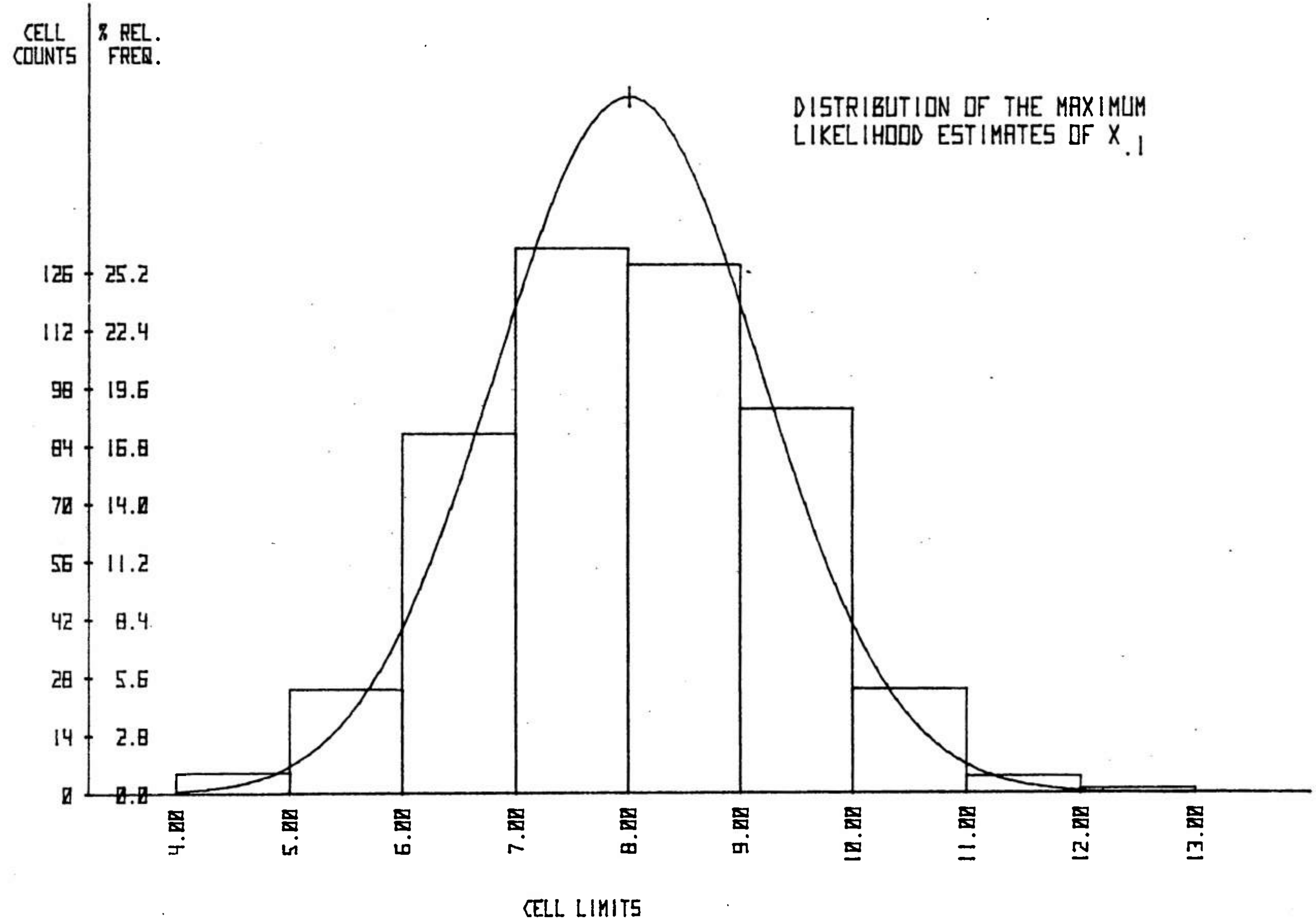
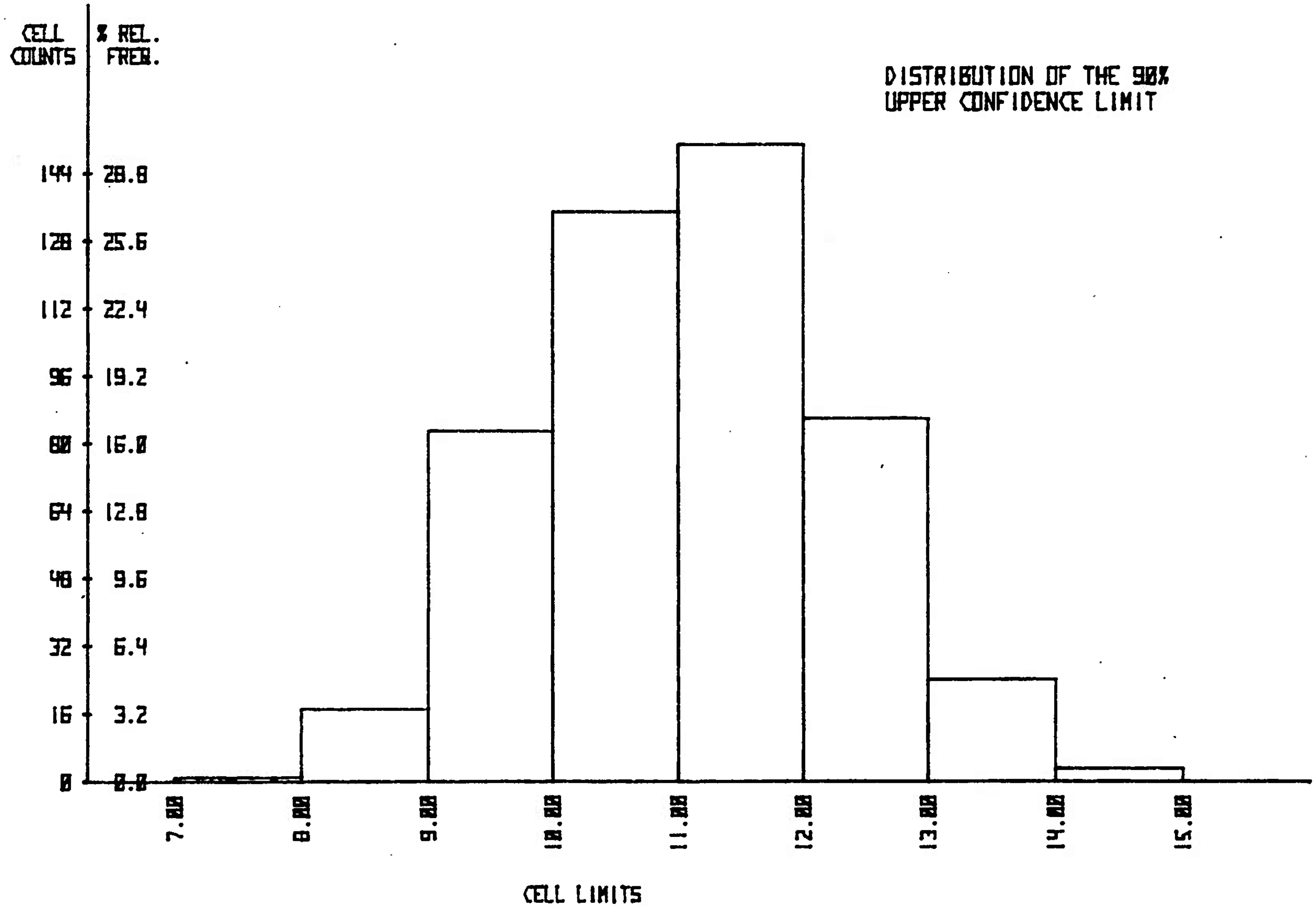

DISTRIBUTION OF THE NUMBER OF ROUNDS REQUIRED
FOR THE MODIFIED ALEXANDER EVD
CELL COUNTS
% REL. FREQ.
64
56
48
40
32
24
16
8
0
12.8
11.2
9.6
8.0
6.4
4.8
3.2
1.6
0.0
45.00
55.00
65.00
75.00
85.00
95.00
105.00
115.00
125.00
135.00
145.00
CELL LIMITS

FIGURE 9



CELL COUNTS
% REL. FREQ.
DISTRIBUTION OF THE MAXIMUM
LIKELIHOOD ESTIMATES OF X .1
126
112
98
84
70
56
42
28
14
0
25.2
22.4
19.6
16.8
14.0
11.2
8.4
5.6
2.8
0.0
4.00
5.00
6.00
7.00
8.00
9.00
10.00
11.00
12.00
13.00
CELL LIMITS

FIGURE 10



DISTRIBUTION OF THE 90%
UPPER CONFIDENCE LIMIT
CELL COUNTS
% REL. FREQ.
144
128
112
96
80
64
48
32
16
0
28.8
25.6
22.4
19.2
16.0
12.8
9.6
6.4
3.2
0.0
7.00
8.00
9.00
10.00
11.00
12.00
13.00
14.00
15.00
CELL LIMITS

FIGURE 11

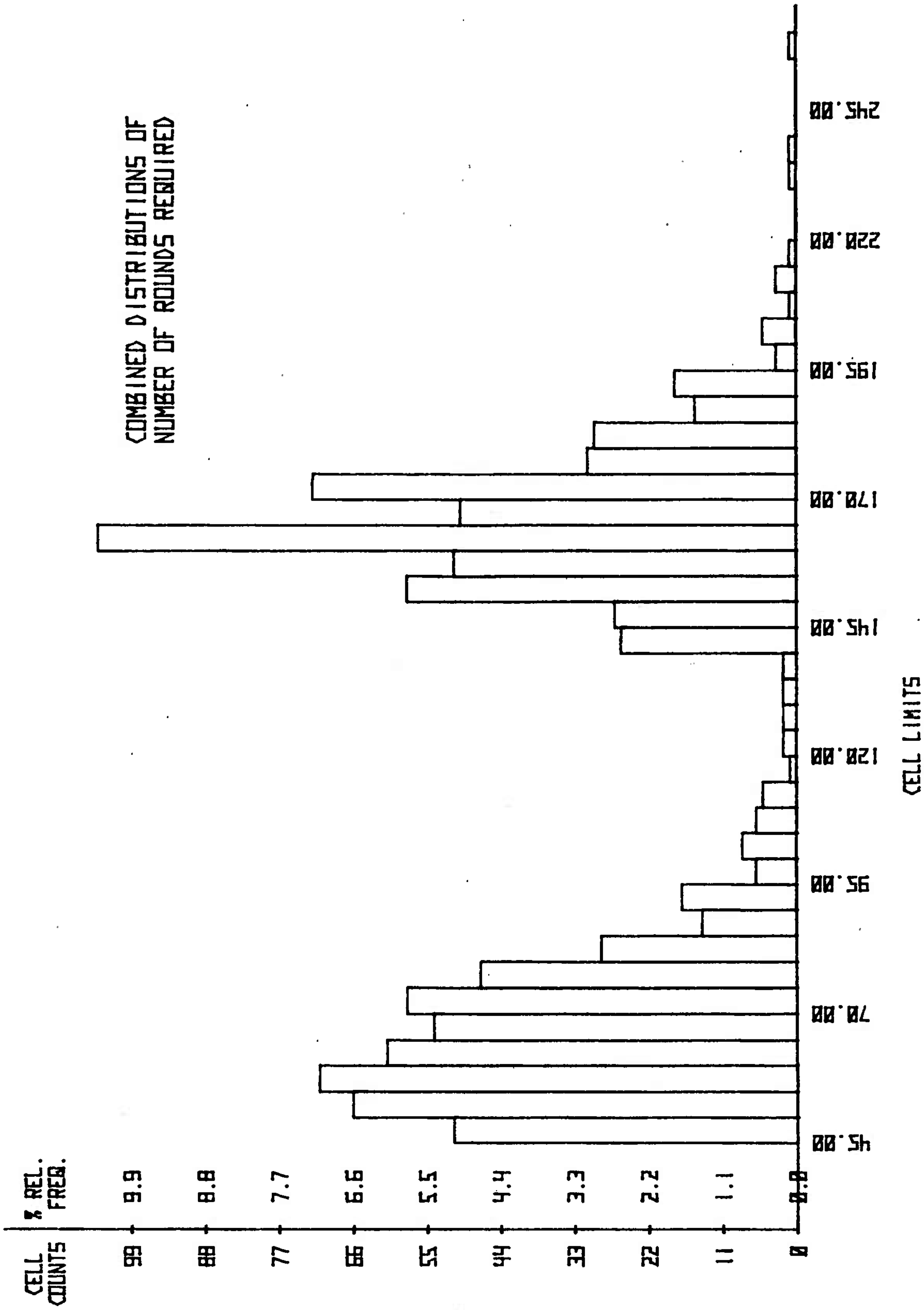

COMBINED DISTRIBUTIONS OF
NUMBER OF ROUNDS REQUIRED
CELL COUNTS
% REL. FREQ.
0
11
22
33
44
55
66
77
88
99
0.0
1.1
2.2
3.3
4.4
5.5
6.6
7.7
8.8
9.9
45.00
70.00
95.00
120.00
145.00
170.00
195.00
220.00
245.00
CELL LIMITS

FIGURE 12

TABLE 1. SUMMARY OF RESULTS*

Distribution Simulated	True $X_{.1}$	Alexander EVD				Modified Alexander EVD				Ratio of Mean Sample Sizes
		Sample Size Mean	Sample Size Range	$\hat{X}_{.1}$	90% Upper C.L.	Sample Size Mean	Sample Size Range	$\hat{X}_{.1}$	90% Upper C.L.	
Normal	7.8	166	134-256	7.8	9.7	67	46-140	8.0	9.6	.41
Exponential	11.4	191	148-324	11.5	17.8	95	46-256	11.9	14.6	.50
Uniform	7.2	154	130-180	7.0	8.9	57	46-91	7.0	7.8	.37
Gamma	9.2	176	134-264	9.2	13.8	80	46-192	9.6	11.7	.45

*500 simulations performed for each distribution using each design.

ACKNOWLEDGEMENT

The authors wish to express their appreciation to Jock O. Grynovicki for his programming help and his time spent in the running of examples on the computer.

APPENDIX

METHOD OF REVERSALS FOR SENSITIVITY DATA

A. Method

The method of reversals is a maximum-likelihood procedure for obtaining distribution-free estimates of a monotone nondecreasing response function. The test stimulus levels are X_i $(i = 1,2,\ldots,k)$ and are ordered from thickest to thinnest shielding thickness,

$$X_1 > X_2 > \ldots > X_k \tag{A.1}$$

If $\hat{p}_i$ is the estimate of the probability of response at X_i, and if we assume that the response function is monotone nondecreasing, then necessarily

$$\hat{p}_1 \leqslant \hat{p}_2 \leqslant \ldots \leqslant \hat{p}_k. \tag{A.2}$$

The algorithm below can be used to find the estimates of the response distribution and their associated stimulus levels.

1) Let X_i $(i = 1,2,\ldots,k)$ be the k stimulus levels at which data have been collected, where $X_1 > X_2 \ldots > X_k$. We wish to find the estimates, $\hat{p}_i$, of the values $p_i = M(X_i)$, the response probabilities at the levels X_i, which satisfy Equation A.2.

2) Let n_i $(i = 1,2,\ldots,k)$ be the number of tests performed at level X_i and f_i $(i = 1,2,\ldots,k)$ be the number of responses observed in the n_i tests. Consider the sequence

$$\frac{f_1}{n_1}, \frac{f_2}{n_2}, \ldots, \frac{f_k}{n_k} .$$

If this sequence is nondecreasing, then the estimates $\hat{p}_i$ are simply given by

$$\hat{p}_i = \frac{f_i}{n_i} .$$

3) If for some i, $\dfrac{f_i}{n_i} > \dfrac{f_{i+1}}{n_{i+1}}$, replace both by

$$\frac{F_{i,i+1}}{N_{i,i+1}} = \frac{f_i + f_{i+1}}{n_i + n_{i+1}} .$$

The new sequence is then

$$\frac{f_1}{n_1}, \frac{f_2}{n_2}, \ldots, \frac{f_{i-1}}{n_{i-1}}, \frac{F_{i,i+1}}{N_{i,i+1}}, \frac{f_{i+2}}{n_{i+2}}, \ldots, \frac{f_k}{n_k} .$$

If this sequence still contains a reversal, a pair of consecutive fractions, for which the first is greater than the second, replace the pair with a single term as above. This process is continued until one obtains a non-decreasing sequence:

$$\frac{\phi_1}{\eta_1}, \frac{\phi_2}{\eta_2}, \frac{\phi_3}{\eta_3}, \ldots,$$

where $\dfrac{\phi_j}{\eta_j} = \dfrac{f_i + \ldots + f_{i+s}}{n_i + \ldots + n_{i+s}}$ for appropriate i and s.

4) The final estimates are given by

$$\hat{p}_i = \ldots = \hat{p}_{i+s} = \frac{\phi_j}{\eta_j} .$$

5) Linear interpolation is used to compute the values of the response function between stimulus levels tested.

B. Example

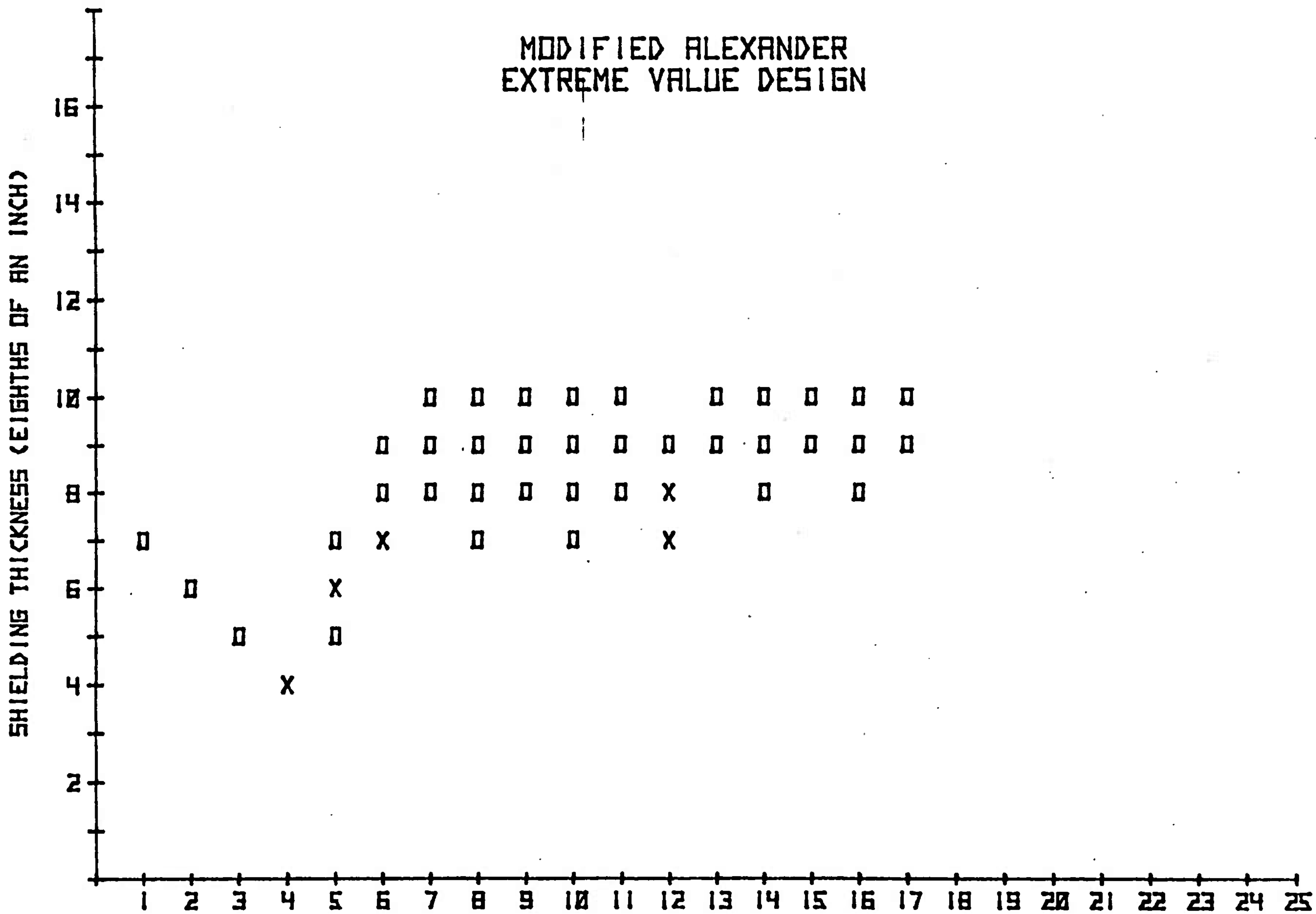
If the results of the experiment were as shown in Figure A1 the maximum likelihood estimate found by the method of reversals is as follows:

Shielding Thickness	f_i/n_i	$\frac{F_{i,i+1}}{N_{i,i+1}}$		$\hat{p}_i$
4/8	1/1			1.0
5/8	0/2	1/4		.3
6/8	1/2		3/10	.3
7/8	2/6			.3
8/8	1/9			.11
9/8	0/12			0
10/8	0/10			0

The shielding thickness corresponding to the .1 quantile is found by linear interpolation.

$$.13\left[\begin{array}{ll} .01\left[\begin{array}{l}8/8 = 1\\ X_{.1}\end{array}\right. & \left.\begin{array}{r}.11\\ .1\end{array}\right] .01 \\ 9/8 = 1.13 & .0\end{array}\right] .11$$

The shielding thickness associated with the .1 quantile is 1.01 inches.



MODIFIED ALEXANDER
EXTREME VALUE DESIGN
SHIELDING THICKNESS (EIGHTHS OF AN INCH)
16
14
12
10
8
6
4
2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
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16
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FIGURE A1

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